Binary Number Systems :-

Book :-
Morris Mano
"Digital Logic & Computer Design"

What is a binary number system?

Everything in computers is stored as 0's and 1's. Each basic memory location called a bit can have two possible states 0 or 1.

Why?

8 bits together from a byte.

form a bigger sequence of data.

0's and 1's are physically stored as low voltage and high voltage in an electronic circuit which we refer to as computer memory.

Binary Numbers :-

10 digits → {0,1,2,3,4,5,6,7,8,9}

decimal numbers → 273

Two hundres, 7 tens, 3 unit

$= 2\times 10^2 + 7\times 10^1 + 3\times 10^0$

$= 2\times10^2 + 7\times10^1 + 3\times10^0$

Base / radix is 10.

$27.32 = 2\times10^1 + 7\times10^0 + 3\times10^{-1} + 2\times10^{-2}$

$= 20 + 7 + \frac{3}{10} + \frac{2}{100}$

Binary multiply by two possible values : 0 and 1.

2^i

Example :-

11010.11

$= 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0 + 1\times2^{-1} + 1\times2^{-2}$

$= 26.75$

In general, a number expressed in base-r system has coefficients multiplied by power of r:-

$$a_n \cdot r^n + a_{n-1} r^{n-1} + \ldots + a_2 \cdot r^2 + a_1 \cdot r + a_0 + a_{-1} r^{-1} + a_{-2} \cdot r^{-2} + \ldots + a_{-m} \cdot r^{-m}$$

Base-5 :-

$(4021.2)_5 = 4\times5^3 + 0\times5^2 + 2\times5^1 + 1\times5^0 + 2\times5^{-1}$

$= (511.4)$

$= (511.4)_{10}$

Numbers with different bases:—

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Any base 'r' to base 10 conversions

→ $(12AB)_{16} = B\times1 + A\times16 + 2\times16^2 + 16^3$

$= (4779)_{10}$

→ $(516.25)_7 = 6\times7^0 + 1\times7^1 + 5\times7^2 + 2\times7^{-1} + 5\times7^{-2}$

$= (\_\_\_\_\_\_)_{10}$.

Conversion from decimal to base-'r'

Integer part

① decimal to binary

$(41)_{10} \rightarrow (\quad)_2$?

division method

integer	remainder
41	
20	1
10	0
5	0
2	1
1	0
0	1

↑ $(101001)_2$

② decimal to octal

$(153)_{10} \rightarrow (\quad)_8$

153	
19	1
2	3
0	2

↑ $(231)_8$

Fractional part

① $(0.6875)_{10} \rightarrow (\quad)_2$

	integer		fraction
$0.6875 \times 2 =$	1	+	0.3780
$0.3750 \times 2 =$	0	+	0.7500
$0.7800 \times 2 =$	1	+	0.5000
$0.5000 \times 2 =$	1	+	0.0000

↓ 1 0 1 1

$\Rightarrow (0.6875) = (0.1011)_2$

$\Rightarrow \quad (0.6875)_{10} = (0.1011)_2$

② $(0.513)_{10} \rightarrow (\quad)_8$

$(0.513) \times 8 = 4.104$

$(0.104) \times 8 = 0.832$

$(0.832) \times 8 = 6.656$

$(0.656) \times 8 = 5.248$

$(0.248) \times 8 = 1.984$

$(0.984) \times 8 = 7.872$

$\Rightarrow \quad (0.513)_{10} = (0.406517\ldots)_8$

Integer and fractional combined:-

$\Rightarrow \quad (41.6878)_{10} \rightarrow (101001.1011)_2$

$(153.513)_{10} \rightarrow (231.406517)_8$