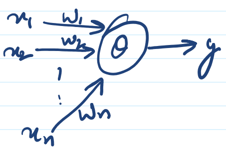
Perceptron

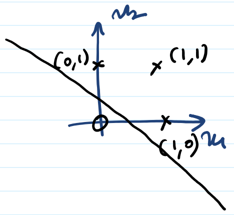
(Rosenblatt 1958)

$$y = \begin{cases} 1, & \text{if } \sum_{i=1}^{n} w_i x_i \geq \theta \\ 0, & \text{otherwise} \end{cases}$$



OR Gate

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1



$w_1(0) + w_2(0) < \theta$
$w_1(0) + w_2(1) \geq \theta$
$w_1(1) + w_2(0) \geq \theta$
$w_1(1) + w_2(1) \geq \theta$

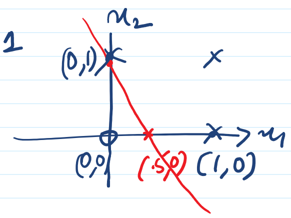
$\Rightarrow$

$\theta > 0$
$w_2 \geq \theta$
$w_1 \geq \theta$
$w_1 + w_2 \geq \theta$

One solution,

$\theta = 1$
$w_1 = 2$
$w_2 = 1$

Solution
$2x_1 + x_2 = 1$



"Many solutions"

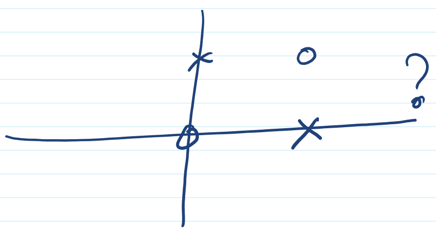
XOR

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$w_1(0) + w_2(0) < \theta$
$w_1(0) + w_2(1) \geq \theta$
$w_1(1) + w_2(0) \geq \theta$
$w_1(1) + w_2(1) < \theta$

$\Rightarrow$

$\theta > 0$
$w_2 \geq \theta$
$w_1 \geq \theta$
$w_1 + w_2 < \theta$



Perceptron: -

n

Perceptron :-

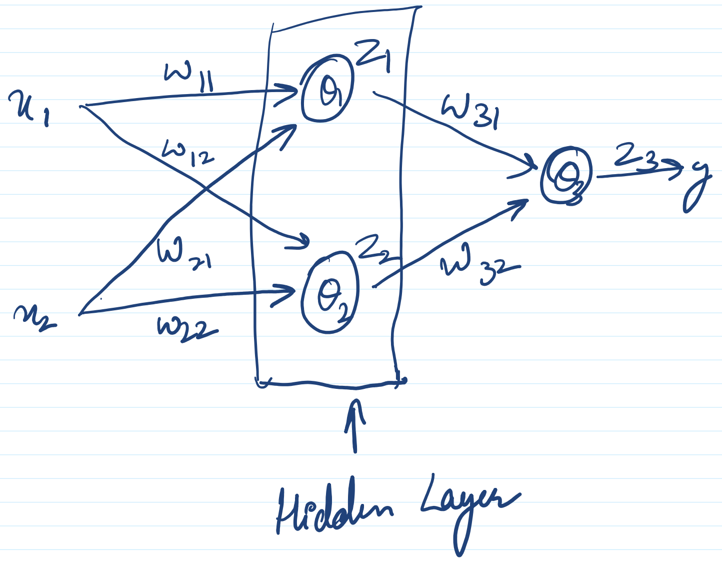
$$y = \begin{cases} 1, & \sum_{i=1}^{n} w_i x_i \geq \theta \\ 0, & \text{otherwise} \end{cases}$$

$$\left.\sum_{i=1}^{n} w_i x_i - \theta \geq 0 \right\} \longrightarrow w^T x \geq 0$$

$$w = [w_0 \; w_1 \; w_2 \; \cdots \; w_n], \quad w_0 = -\theta = b$$

$$x = [x_0 \; x_1 \; x_2 \; \cdots \; x_n], \quad x_0 \to 1$$

Hidden Layer :-



x_1	x_2	$\sum_{j=1}^{2} w_{1j} x_j$	$\sum_{j=1}^{2} w_{2j} x_j$	z_1	z_2	$\sum_{j=1}^{2} w_{3j} z_j$	$y = z_3$
0	0	0	0	0	0	0	0
0	1	-1	2	0	1	2	1
1	0	2	-1	1	0	2	1
1	1	1	1	0	0	0	0

A possible choice of weights and biases. $\quad -2x_1 - x_2 = 1.5$

$-x_1 + x_2 = 1.5$

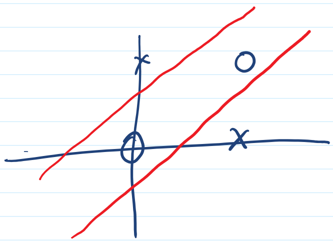
$w_{11} = 2 \qquad w_{21} = -1 \qquad w_{31} = 2$

$w_{12} = -1 \qquad w_{22} = 2 \qquad w_{32} = 2$

$\theta_1 = 1.5 \qquad \theta_2 = 1.5 \qquad \theta_3 = 1$

$2x_1 - x_2 = 1.5$

$-x_1 + 2x_2 = 1.5$



THEOREM :-

Any boolean function of 'n' inputs can be represented exactly by a network of perceptrons containing 1 hidden layer with 2^n perceptrons and output layer containing 1 perceptron.