Mid-sem Answer Key

03 March 2024 12:57

1. (a) Rotation of 3-points along x-axis

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

Given $\phi = 30^\circ$.

Translation vector (T) $= \begin{bmatrix} x_t \\ y_t \\ z_t \end{bmatrix} = \begin{bmatrix} 15 \\ 20 \\ 10 \end{bmatrix}$

4x4 Affine matrix

$$A = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 & 15 \\ 0 & \sqrt{3}/2 & -1/2 & 20 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ & & & 1 \end{bmatrix}$$

(b) Origin $\rightarrow (0,0,0)$

$$\Rightarrow X = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Transformation, $AX = \begin{bmatrix} 1 & 0 & 0 & 15 \\ 0 & \sqrt{3}/2 & -1/2 & 20 \\ 0 & 1/2 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 15 \end{bmatrix}$

$$= \begin{bmatrix} 15 \\ 20 \\ 10 \\ 1 \end{bmatrix}$$

$\Rightarrow$ $(0,0,0)$ goes to $(15, 20, 10)$

(c) $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

$|A| = 21 - 20 = 1$

$\text{adj } A = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

$A^{-1} = \dfrac{\text{adj } A}{|A|} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$

(d) Flood - fill Polygon Filling :-

2. (a) $(x_1, y_1) = (1, 1)$

$(x_2, y_2) = (8, 5)$

$dx = \quad x_2 - x_1 = 7$

$dy = \quad y_2 - y_1 = 4$

$\Rightarrow$ decision parameter $(p) = 2dy - dx$

$= 8 - 7$

$= 1$

Initial point (x, y)

if $\quad P < 0$

next point $(x+1, y)$

$$P_K = P_{K-1} + 2dy$$

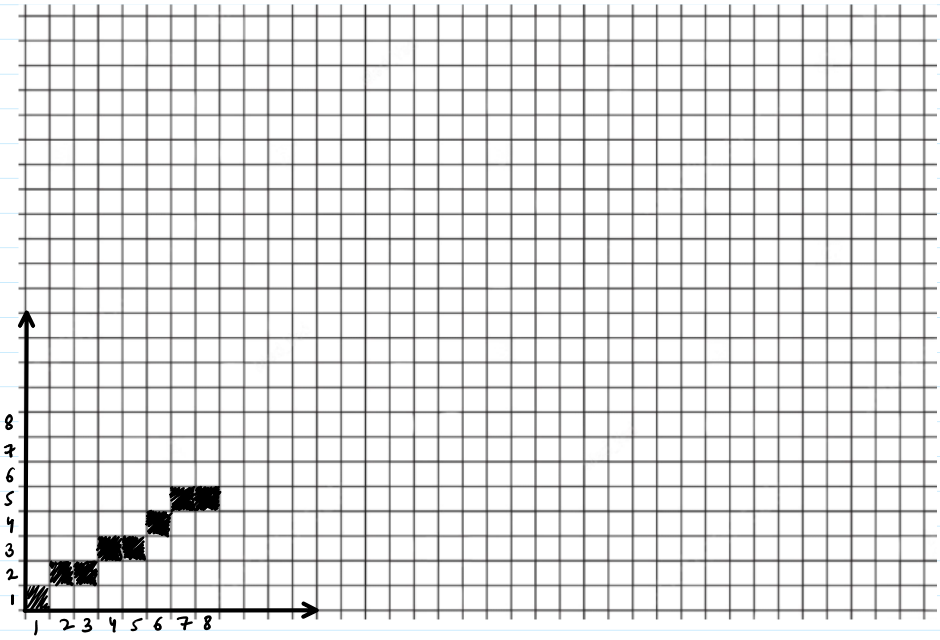
if $\quad P \geq 0$

next point $(x+1, y+1)$

$$P_K = P_{K-1} + 2dy - 2dx$$

p	x	y
1	1	1
-5	2	2
3	3	2
-3	4	3
5	5	3
-1	6	4
7	7	4
	8	5

7	7 8	



3. (b)

$$P = \left[\begin{array}{ccc|c} -9 & 2 & 3 & 1 \\ 3 & -9 & 6 & 1 \\ 2 & 6 & -10 & 1 \end{array}\right] = \left[\underbrace{\begin{matrix} -9 & 2 & 3 \\ 3 & -9 & 6 \\ 2 & 6 & -10 \end{matrix}}_{M} \;\; \underbrace{\begin{matrix} 1 \\ 1 \\ 1 \end{matrix}}_{p_4}\right]$$

$$\text{Camera Centre } (C) = -M^{-1} p_4$$

$$\text{Cofactor}(M) = \begin{bmatrix} 54 & 42 & 36 \\ 38 & 84 & 58 \\ 39 & 63 & 75 \end{bmatrix}$$

$$\det(M) = -294$$

$$M^{-1} = \frac{-1}{294} \begin{bmatrix} 54 & 42 & 36 \\ 38 & 84 & 58 \\ 39 & 63 & 75 \end{bmatrix}$$

Camera Centre $(\tilde{C}) = \frac{1}{294}\begin{bmatrix}131\\189\\169\end{bmatrix}$

(b) Vanishing point of x-axis:

$$P[1\ 0\ 0\ 0]^T = p_1$$

$$\left(-9/2,\ 3/2\right)$$

(c) Image point of origin:

$$P[0\ 0\ 0\ 1]^T = p_4$$

$$(1, 1)$$

(c) 1. To keep one point $P(h,k)$ fixed, first translate (h,k) to the origin, apply scaling and then translate back to (h,k).

$$\begin{bmatrix}1 & 0 & h\\0 & 1 & k\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}s_x & 0 & 0\\0 & s_y & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & -h\\0 & 1 & -k\\0 & 0 & 1\end{bmatrix}$$

$$= \begin{bmatrix} s_x & 0 & -h s_x + h \\ 0 & s_y & -k s_y + k \\ 0 & 0 & 1 \end{bmatrix}$$

2. Transformation matrix,

$$T = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Points in homogeneous coordinates

$$A = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

Magnified Points using T

$$A' = TA = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -2 \\ 1 \end{bmatrix}$$

$$B' = TB = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$$

$$C' = TC = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 1 \end{bmatrix}$$

$\Rightarrow$ New points $(-5, -2)$ $(-3, 0)$ $(5, 2)$