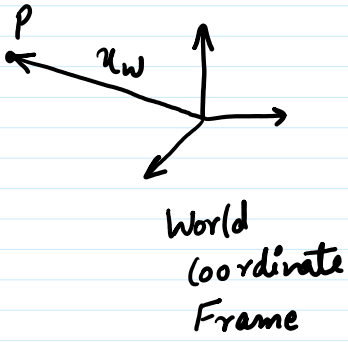
9. Perspective Projection

07 February 2024 12:28



Camera lies on the world coordinate frame.

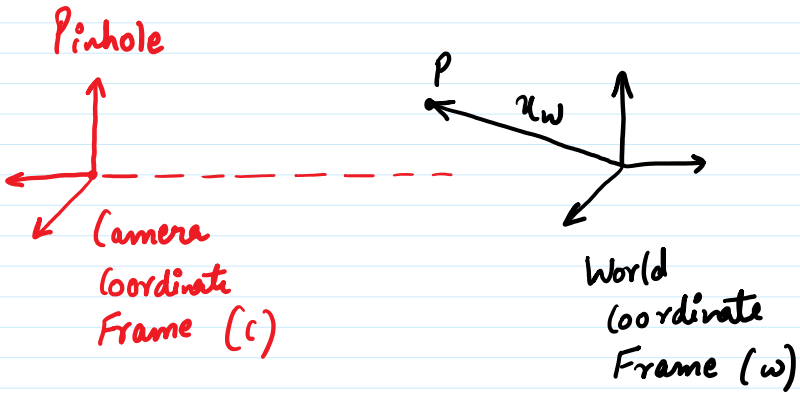
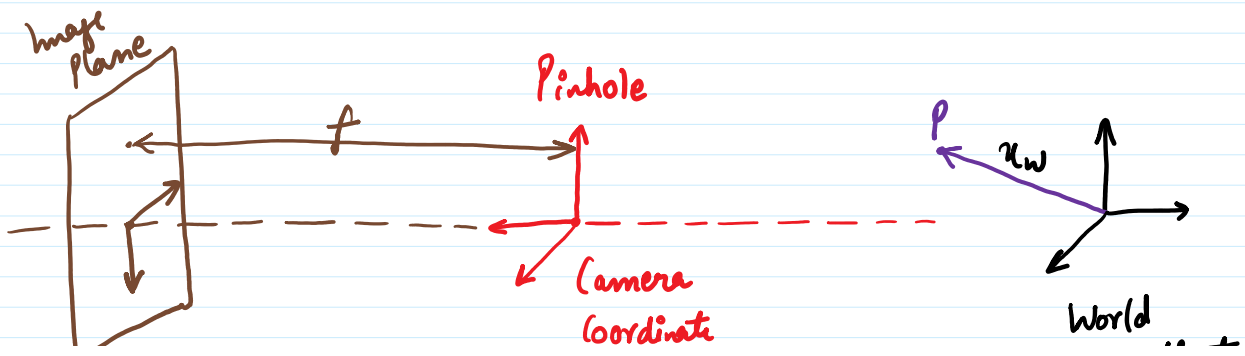


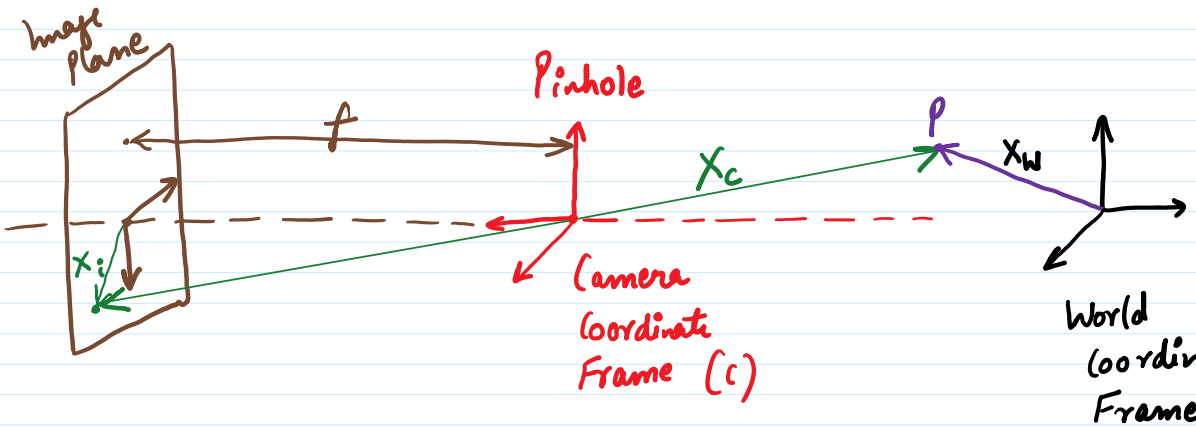
Image Plane is at a distance of 'f' from the camera frame 'c'
This distance is called as "Focal Length (f)"



↙ Camera Coordinate Frame (c)

↙ World coordinate Frame (w)

The goal is to know the relative position of 'c' wrt 'w' to take from point P in 'w' to point x_i in the image plane.



$$X_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Image Coordinates

$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

Camera Coordinates

$$X_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

World Coordinates

Steps in 3D to 2D Imaging Model:-

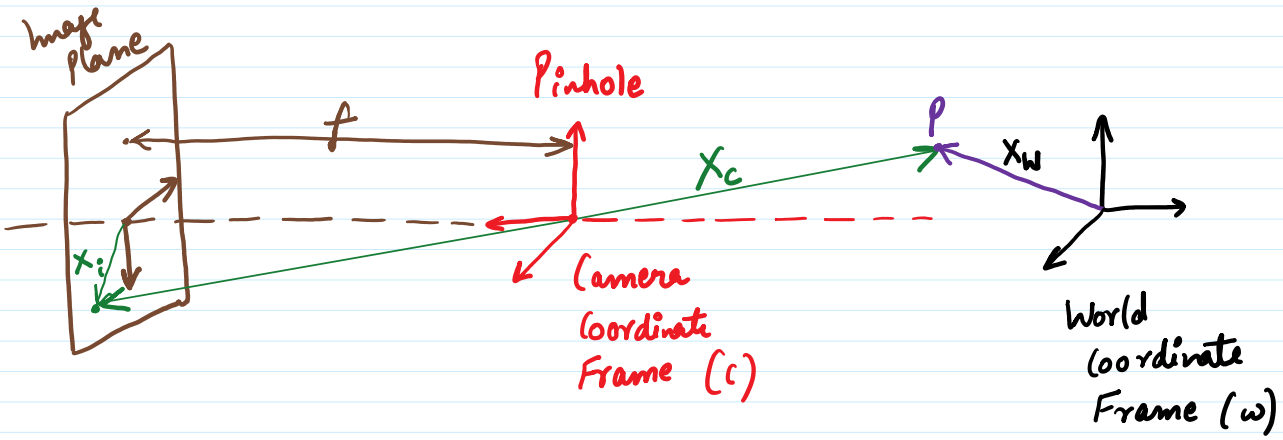
① Coordinate Transformation

World Coordinates → Camera Coordinates

② Perspective Projection

Camera Coordinates → Image Coordinates

Perspective Projection :-



$$X_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Image Coordinates

$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$$

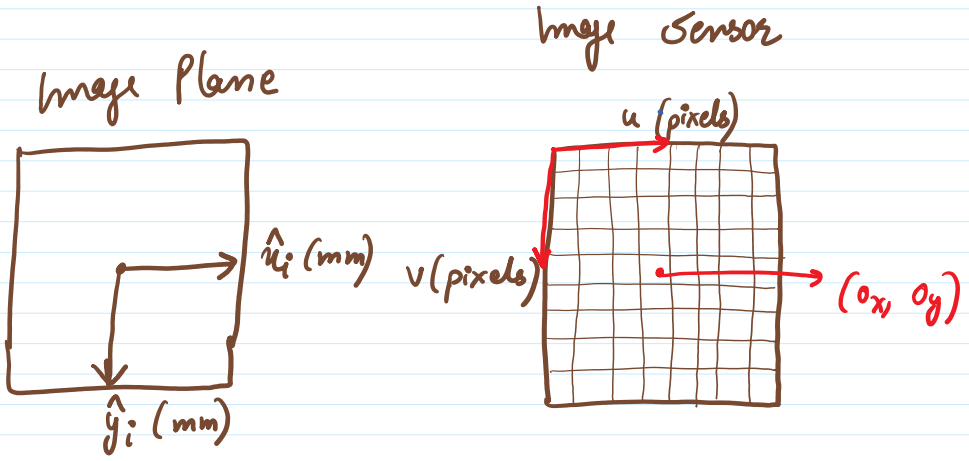
Camera Coordinates

From diagram,

$$\frac{x_i}{f} = \frac{x_c}{z_c} \quad \text{and} \quad \frac{y_i}{f} = \frac{y_c}{z_c}$$

$$\Rightarrow x_i = f\frac{x_c}{z_c} \quad \text{and} \quad y_i = f\frac{y_c}{z_c} \quad \text{—— (1)}$$

where (x_i, y_i) are the coordinates of points on the image.



If m_x and m_y are the pixel densities (pixels/mm) in x and y directions,

$\Rightarrow$ Top-left corner is origin.

$\Rightarrow$ (o_x, o_y) is the principle point where optical axis pierces.

Then Pixel coordinates becomes:

from equation ①,

$$\left.\begin{aligned} u = m_x x_i &= m_x f \frac{x_c}{z_c} + o_x \\ v = m_y y_i &= m_y f \frac{y_c}{z_c} + o_y \end{aligned}\right\} \text{— ②}$$

pixel density focal length

Pixel density and focal length are unknown.
are properties of the camera.

Let $f_x = m_x f$

$f_y = m_y f$

$\Rightarrow$ Put in equation (2)

$$u = f_x \frac{x_c}{z_c} + O_x, \quad v = f_y \frac{y_c}{z_c} + O_y$$

$$u = f_x \frac{x_c}{z_c} + O_x, \quad v = f_y \frac{y_c}{z_c} + O_y$$

4 unknowns

(f_x, f_y) → focal length in x and y direction.

(O_x, O_y) → Principle point.

(f_x, f_y, O_x, O_y) → Intrinsic Parameters of the camera.

"Camera's Internal Geometry"