$$P(t) = \sum_{i=0}^{n} b_i J_i^n(t)$$

$$\boxed{J_i^n(t) = {}^nC_i t^i (1-t)^{n-i}}$$

$$J_0^n(0) = {}^nC_0 \, 0^0 (1-0)^n$$
$$= 1 \cdot 1 \cdot 1$$
$$= 1$$

$$\frac{n!}{0!\,n!}$$

$$\boxed{\begin{array}{l} {}^nC_0 = 1 \text{ because } 0! = 1 \\ x^0 = 1 \end{array}}$$

Properties :–

① End point Interpolation

At $t=0$

$$i=0,\ J_0^n(0) = \underbrace{{}^nC_0}_{1} \underbrace{(0)^0}_{1} \underbrace{(1-0)^n}_{1} = 1$$

$$i \neq 0,\ J_i^n(0) = {}^nC_i (0)^i (1-0)^n = 0$$

$$P(0) = b_0 J_0^n(0) = b_0$$

At $t=1$

$$i=n,\ J_n^n(1) = \frac{n!}{n!(1)} (1)^n (0)^{n-n} = 1$$

$$i \neq n,\ J_i^n(1) = \frac{n!}{(n-i)!\,i!} (1)^n (1-1)^{n-i} = 0$$

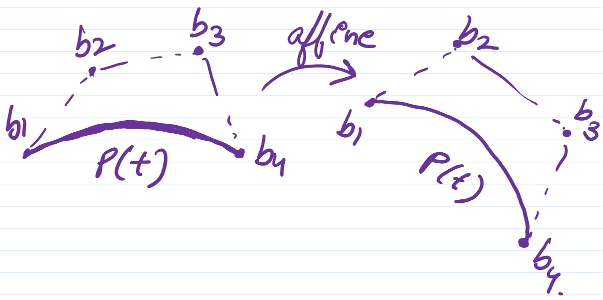
$$P(1) = b_n J_n^n(1) = b_n$$

$$P(t) = \sum_{i=0}^{n} b_i J_i^n(t)$$

$$\boxed{J_i^n(t) = {}^nC_i t^i (1-t)^{n-i}}$$

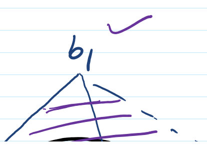
② Affine Invariance

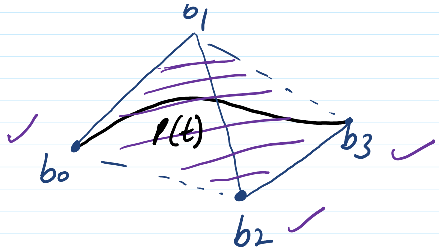
Applying an affice transfomation to the curve is equivalent to applying the transformation to the control points.



③ Convex hull

Curve lies in the convex hull of the control points





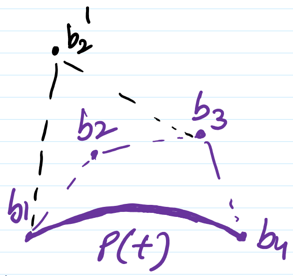
$$\sum_{i=0}^{n} J_i^n(t) = 1$$

$J_i^n(t)$: non-negative for $t \in [0,1]$

(4) Symmetry

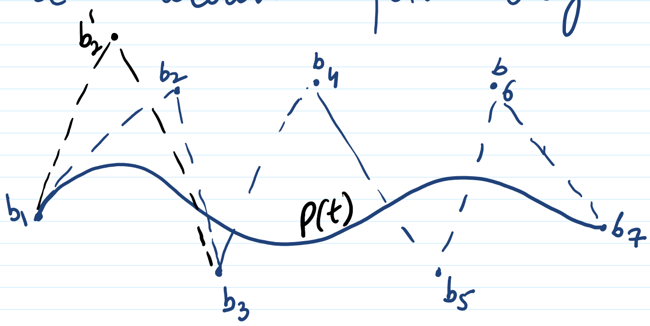
$P(t)$ defined by $b_0, b_1, b_2, \dots, b_n$ is equal to $P(1-t)$ defined by $b_n, b_{n-1}, \dots, b_0$

$$\sum_{i=0}^{n} b_i J_i^n(t) = \sum_{i=0}^{n} b_{n-i} J_i^n(1-t)$$



(5) Pseudo Local Control

Moving a Bezier point the whole curve will change but the maximum influence will be around point only.

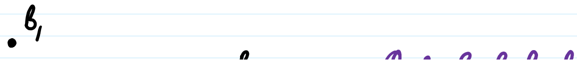


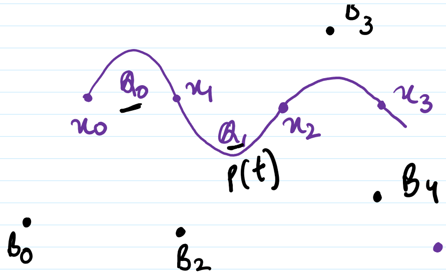
B_Splines

Each control point is associated with a unique basis / blending function.

Each point affects the shape of the curve over a range of parameter values where the basis function is non-zero.

$\Rightarrow$ "LOCAL CONTROL"





$Q_0 : B_0 B_1 B_2 B_3$

$Q_1 : B_1 B_2 B_3 B_4$

• knot point

• Control point

Parameter t is defined as

$x_i \le t \le x_{i+1}$

$x_0\ x_1\ x_2\ x_3$: knot values.

Mathematically,

polynomial spline function,

$$P(t) = \sum_{i=0}^{n} B_i \underbrace{N_{i,k}(t)}_{\uparrow} \qquad t_{min} \le t \le t_{max}$$

$$2 \le k \le n+1$$

where

$N_{i,k}$ is blending/basis function

$$N_{i,1}(t) = \begin{cases} 1 & x_i \le t < x_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,k}(t) = \frac{(t - x_i)\, N_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t)\, N_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$

Properties :–

① Local control

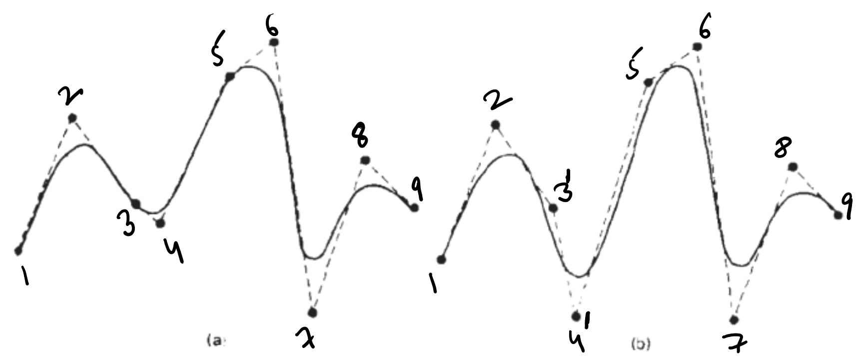


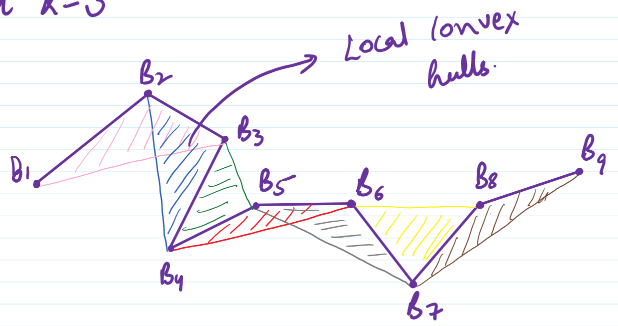
Figure 10-41
Local modification of a B-spline curve. Changing one of the control points in (a) produces curve (b), which is modified only in the neighborhood of the altered control point.

② Convex Hull

For a B-Spline curve of order K a point on the curve lies within the convex hull of K neighboring points.

All points of B-Spline curve must lie within the union of all such convex hulls.

Let $K=3$



BSpline surfaces :-

$$P(u,v) = \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} B_{i_1,i_2} \, N_{i_1,K_1}(u) \, N_{i_2,K_2}(v)$$

Summary :-

	Cubic Splines	Hermite Splines	Bezier Curves	B-Spline
Interpolation	All control points	End Point	End Point	No
Local Control	No	Yes	No	Yes