Perspective Projection :-

$$\boxed{u = f_x \frac{x_c}{z_c} + o_x, \quad v = f_y \frac{y_c}{z_c} + o_y}$$

Using homogeneous representation,

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} z_c u \\ z_c v \\ z_c \end{bmatrix} = \begin{bmatrix} f_x x_c + z_c o_x \\ f_y y_c + z_c o_y \\ z_c \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

"Linear Model for Perspective Projection"

Intrinsic Matrix :-

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

Calibration Matrix :

$$k = \begin{bmatrix} f_x & 0 & o_x \\ 0 & f_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

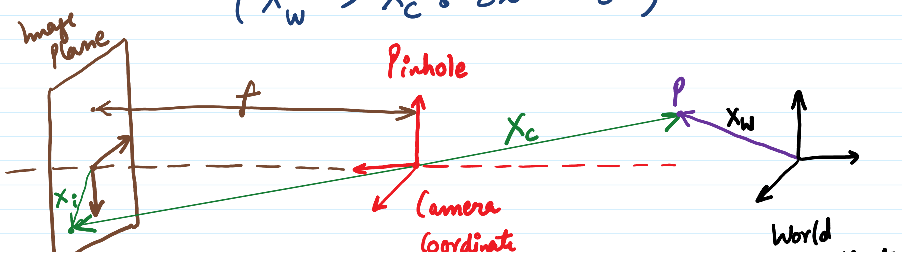
upper right triangular matrix

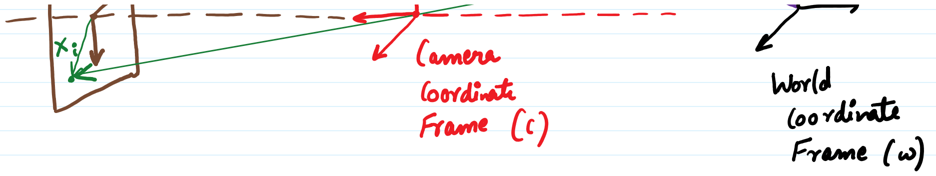
Intrinsic Matrix :

$$M_{int} = [k|0] = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Coordinate Transformation :-

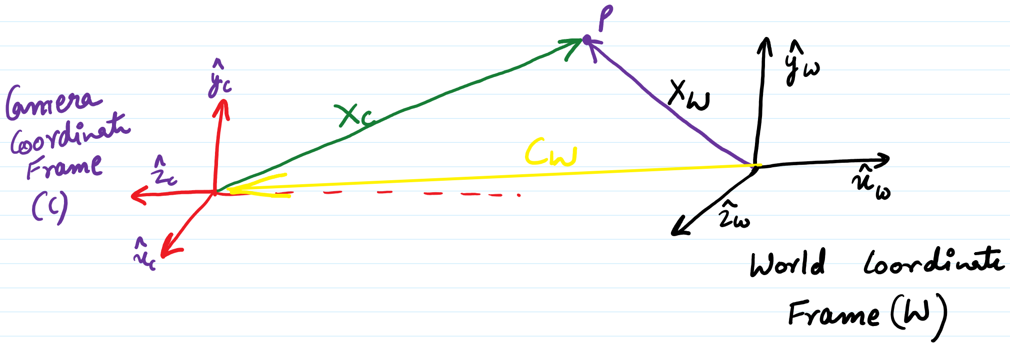
$(X_w \rightarrow X_c : 3D \rightarrow 3D)$





$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} \qquad X_w = \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix}$$

Camera Coordinates   World Coordinates



Position C_W and orientation R of the camera in the world coordinate frame (W) are the camera's Extrinsic Parameters.

$$\underbrace{R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{33} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}}_{\text{orthogonal matrix}} \begin{array}{l} \to \text{Row 1: Direction of } \hat{x}_c \text{ in world coordinate frame.} \\ \to \text{Row 2: ——— } \hat{y}_c \text{ ———————.} \\ \to \text{Row 3: ——— } \hat{z}_c \text{ ———————.} \end{array}$$

Given (R, C_w) of the camera, the location of point P in the world coordinate frame (w).

$$X_c = R(X_w - C_w)$$

$$= R X_w - R C_w = R X_w + t \qquad \boxed{t = -R C_w}$$

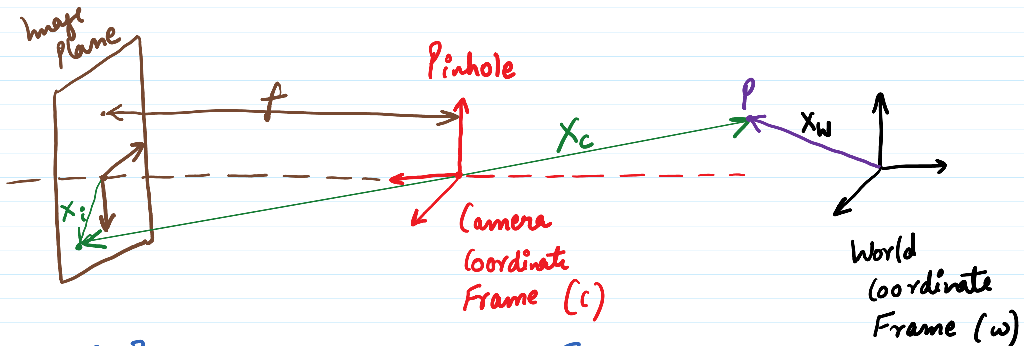
$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Using homogeneous coordinates :-

$$X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Extrinsic Matrix (M_{ext}) $= \begin{bmatrix} R_{3\times3} & t \\ 0_{1\times3} & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$



$X_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$ Image coordinates $\xleftarrow{M_{int}}$ Perspective Projection $\quad X_c = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix}$ Camera Coordinates $\xleftarrow{M_{ext}}$ Coordinate Transformation $\quad X_W = \begin{bmatrix} x_W \\ y_W \\ z_W \end{bmatrix}$ World coordinates

Camera to Pixel

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

$$\boxed{u = M_{int} X_c}$$

World to Camera

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

$$\boxed{X_c = M_{ext} X_W}$$

Combining full projection matrix P:

$$\boxed{\begin{aligned} u &= M_{int} M_{ext} x_w \\ u &= P x_w \end{aligned}}$$

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}_{3 \times 4}}_{P \text{ (Projection Matrix)}} \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$